

Indian Statistical Institute, Bangalore Centre.
Backpaper Exam : Graph Theory

Instructor : Yogeshwaran D.

Date : July 11th, 2016.

Max. points : 50.

Time Limit : 3 hours.

Give necessary justifications and explanations for all your arguments. If you are citing results from the class, mention it clearly. Results from assignments need to be proved. Answer any five questions. Only the first five answers will be evaluated.

- (a) Let \mathcal{Q}_n be the hypercube graph on n vertices. What is the vertex-Connectivity ($\kappa(\mathcal{Q}_n)$) and edge-connectivity($\lambda(\mathcal{Q}_n)$) of this graph? (5)
(b) A graph G is k -connected if and only if the join $G \vee K_r$ is $(k+r)$ -connected(5)
- Let F be a set of edges in G . Prove that F is an edge-cut in G iff F contains an even number of edges from every cycle C . (10)
- A 3-regular simple graph G has a 1-factor iff it decomposes into copies of P_4 . (10)
- Let M, M' be minimal spanning trees of the graph G with edge-weights $w(\cdot)$. Show that for any $s \geq 0$,

$$|\{e \in M : w(e) = s\}| = |\{e \in M' : w(e) = s\}|. \quad (10)$$

- Compute the eigenvalues of the Laplacian and Adjacency matrices of the the complete graph. Compute the eigenvalues of the adjacency matrix of the complete bi-partite graph. (10)
- Two people play a game on a graph G alternatively choosing distinct vertices. Player 1 starts by choosing any vertex. Each subsequent choice must be adjacent to the preceding choice of the other player. Thus together, the two player are creating a path in the graph. The

last player able to move wins the game. Show that the second player has a winning strategy if the graph G has a perfect matching and otherwise the first player has a winning strategy.